The Chain Rule (Continued)

1. $h(x) = [g(x)]^n$

$$h'(x) = n\,[g(x)]^{n-1} \cdot g'(x).$$

$$h(x) = \sin^n x = (\sin x)^n \Rightarrow h'(x) = n \sin^{n-1} x \cdot \cos x.$$

2. $h(x) = g(mx)$

$$h'(x) = g'(mx) \cdot [mx]' = g'(mx) \cdot m$$

$$h(x) = \sin 3x \Rightarrow h'(x) = \cos(3x) \cdot 3$$

Eg.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
$1/4$	2	$4/3$	16	$8/5$
0	-1	$\pi/6$	4	1
2	0	3	0	5

Say $\left.\begin{aligned} h(x) &= f\left(\sin^2(g(x))\right) \\ k(x) &= g\left(1 + [f(x)]^2\right) \end{aligned}\right\}$ Find $h'(0)$ & $k'(0)$.

$$h(x) = \overset{1}{f}\left[\left\{\overset{3}{\sin}\left(\underset{4}{g(x)}\right)\right\}^{\overset{2}{2}}\right]$$

$$h'(x) = f'\left[\left\{\sin(g(x))\right\}^2\right] \cdot 2\sin(g(x)) \cdot \cos(g(x)) \cdot g'(x)$$

At $x = 0$

$$h'(0) = f'\left[\{\sin(g(0))\}^2\right] \cdot 2\sin(g(0)) \cdot \cos(g(0)) \cdot g'(0)$$

$$= f'\left[(\sin \pi/6)^2\right] \cdot 2\sin(\pi/6) \cdot \cos(\pi/6) \cdot 1$$

$$= f'\left[\left(\tfrac{1}{2}\right)^2\right] \cdot 2 \cdot \left(\tfrac{1}{2}\right) \cdot \left(\tfrac{\sqrt{3}}{2}\right) \cdot 1$$

$$= f'\left(\tfrac{1}{4}\right) \cdot \frac{\sqrt{3}}{2}$$

$$= 16 \cdot \frac{\sqrt{3}}{2} = 8\sqrt{3}.$$

$$k(x) = g\left(1 + [f(x)]^2\right)$$

$$k'(x) = g'\left(1 + [f(x)]^2\right) \cdot \left(0 + 2f(x)\right) \cdot f'(x)$$

$$k'(0) = g'\left(1 + [f(0)]^2\right) \cdot \left(2f(0)\right) \cdot f'(0)$$

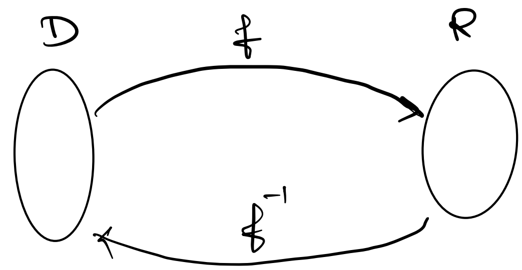
$$= g'\left(1 + (-1)^2\right) \cdot 2(-1) \cdot 4$$

$$= -8\, g'(2)$$

$$= -8(5) = -40$$

Recall:

Inverse Trigonometric Functions:



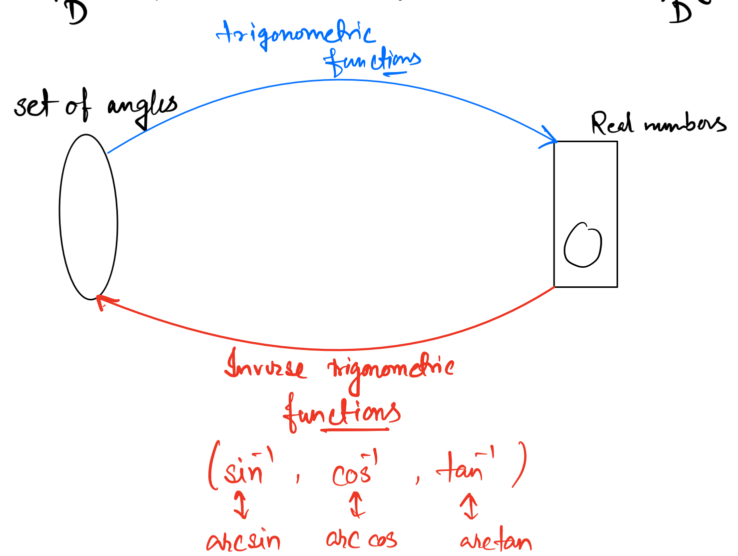
$f(f^{-1}(y)) = y \iff (f \circ f^{-1})(y) = y = id_R(y) \iff f \circ f^{-1} = id_R$

$f^{-1}(f(x)) = x \iff (f^{-1} \circ f)(x) = x = id_D(x) \iff f^{-1} \circ f = id_D$

identity maps –

Say your domain & range both are same as D

$id_D : D \longrightarrow D$ such that $id_D(x) = x$.



Therefore, $\sin(\sin^{-1}(x)) = x$, $\forall x \in \mathbb{R}$ & $\sin^{-1}(\sin\theta) = \theta$, $\forall \theta$

Find the derivative of $\sin^{-1}x$

$$\sin(\sin^{-1}(x)) = x$$

Take derivative with respect to x (wrt x) on bothside

$$\cos(\sin^{-1}(x)) \cdot [\sin^{-1}(x)]' = \frac{d}{dx}(x) = 1.$$

$$\Rightarrow [\sin^{-1}(x)]' = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}}$$

$\sin^2\theta + \cos^2\theta = 1$ for any θ

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$$, our $\theta = \sin^{-1}(x)$

$$\cos(\sin^{-1}(x)) = \sqrt{1-[\sin(\sin^{-1}(x))]^2}$$

$$= \sqrt{1-(x)^2}$$

$$= \sqrt{1-x^2}$$

Formulas for Inverse Trigonometric Functions:

when $f(x) = x$

$$\frac{d}{dx}\left[\sin^{-1}(f(x))\right] = \frac{f'(x)}{\sqrt{1-\{f(x)\}^2}} \qquad \frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}(f(x))\right] = -\frac{f'(x)}{\sqrt{1-\{f(x)\}^2}} \qquad \frac{d}{dx}\left(\cos^{-1}x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}(f(x))\right] = \frac{f'(x)}{1+\{f(x)\}^2} \qquad \frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\cot^{-1}(f(x))\right] = -\frac{f'(x)}{1+\{f(x)\}^2} \qquad \frac{d}{dx}\left(\cot^{-1}x\right) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1}(f(x))\right] = \frac{f'(x)}{f(x)\cdot\sqrt{\{f(x)\}^2-1}} \qquad \frac{d}{dx}\left(\sec^{-1}x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left[\csc^{-1}(f(x))\right] = -\frac{f'(x)}{f(x)\cdot\sqrt{\{f(x)\}^2-1}} \qquad \frac{d}{dx}\left(\csc^{-1}x\right) = -\frac{1}{x\sqrt{x^2-1}}$$